

Photon Gas Thermodynamics in Doubly Special Relativity

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Abstract

Doubly special relativity (DSR), with both an invariant velocity and an invariant length scale, elegantly preserves the principle of relativity between moving observers, and appears as a promising candidate of the quantum theory of gravity. We study the modifications of photon gas thermodynamics in the framework of DSR with an invariant length $|\lambda|$, after properly taking into account the effects of modified dispersion relation, upper bounded energy-momentum space, and deformed integration measure. We show that with a positive λ , the grand partition function, the energy density, the specific heat, the entropy, and the pressure are smaller than those of special relativity (SR), while the velocity of photons and the ratio of pressure to energy are larger. In contrast, with a negative λ , the quantum gravity effects show up in the opposite direction. However, these effects only manifest themselves significantly when the temperature is larger than $10^{-3}E_p$. Thus, DSR can have considerable influence on the early universe in cosmological study.

Key words: doubly special relativity, photon gas, thermodynamics

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1 Introduction

Some developments of quantum gravity (QG) suggest a smallest length scale for the structure of space-time, or equivalently, an upper energy bound for particles in the quantum geometrical background [1,2,3,4,5,6,7,8,9,10,11]. The most natural candidate for the minimal length appears to be the Planck length $l_P \equiv \sqrt{G\hbar/c^3} \simeq 1.6 \times 10^{-35}$ m (or correspondingly, the Planck energy, $E_P \equiv \sqrt{\hbar c^5/G} \simeq 1.22 \times 10^{19}$ GeV, for the maximal energy of particles). These typical constants can arise from the combination of the quantum (\hbar), the relativity (c), and the gravity (G).¹ In such scenarios, the Lorentz symmetry of space-time breaks down at quantum-gravitational scale, and this might leave “relic” effects at relatively lower energies and modify low energy physics with extra terms [13,14,15,16].

However, there arises an apparent puzzle — it is well-known that in the special relativity (SR), the length of an object transforms between two observers with relative movements according to Lorentz-Fitzgerald contraction, so in whose reference frame is the smallest length scale which is mentioned above measured? This problem is deeply related to an essential property of physical laws among inertial frames, the so-called principle of relativity, which is regarded as a milestone in the progress of physics. However, in the domain of quantum gravity physics, it is not obvious that this principle still holds firmly. Instead, it deserves the most careful contemplations.

Amelino-Camelia *et al.* suggested a way to reconcile the paradox between the relativity and the minimal length [1,2,3,4,5,6,7,8,9,10,11]. There are two constants that are preserved in such a theory, so it is usually called “doubly special relativity” (DSR) [2,9]. DSR preserves the relativity between inertial frames, whereas deforms the Lorentz algebra with nonlinear actions as a cost. Consequently, the well-known dispersion relation for a particle in SR, $E^2 = p^2 + m^2$, has to be modified in the form with extra QG imprints,

$$E^2 = p^2 + m^2 + \eta E^3 + \dots, \quad (1)$$

where η is a parameter believed to be of the order of the Planck length. Meanwhile the laws of transforming energy and momenta between different inertial observers are also explicitly deformed in DSR. Hence it is possible that a single energy or momentum scale is invariant. We should emphasize that modified Lorentz transformations in momentum space can have physically substantial effects, even though the modifications for different kinds of particles are identi-

¹ However, there are also arguments that a new fundamental scale might appear rather than the conventional Planck scale, see *e.g.*, Ref. [12] and references therein.

cal, since the departure from SR can manifest its effects when we measure and compare physical observables in different conditions (for detailed discussions, see *e.g.*, Ref. [9]).

It is interesting that such modifications lead to many further predictions which can be tested in a series of experiments. One serious example is the energy-dependence of light speed, and its effect is relevant to γ -ray burst observations [17,18,19,20,21,22,23], some corrections to the predictions of inflationary cosmology [24] and dark energy [25]. Indeed, although some theoretical calculations motivate the modification of the dispersion relation, the fact that a modified dispersion relation is experimentally measurable by itself is a more exciting reason to take it into serious consideration.

As pointed out in Refs. [4,26,27], the law of composition of momenta in SR has to be modified together with the modification of the dispersion relation, since the theory is relativistic with two invariant quantities, and accordingly the role of integration over energy-momentum space is modified. There are some studies [28,29] on the effects from the modified dispersion relation on a photon gas. In our paper, we study the modified thermodynamics of the photon gas in the framework of DSR, with more careful considerations compared to previous studies. We properly include the effects of the modified dispersion relation, the deformed integration measure, and the upper energy-momentum bound. We find that the number of available microstates is modified and consequently thermodynamical quantities are altered in DSR. We show that with a positive parameter λ , the grand partition function, the energy density, the specific heat, the entropy, and the pressure of the photon gas are smaller than those of SR, while the velocity of photons and the ratio of pressure to energy are larger. In contrast, with a negative λ , the quantum gravity effects show up in the opposite direction. However, these effects only manifest themselves significantly when the temperature is larger than $10^{-3}E_P$.

The paper is organized as follows. In Sec. 2, we review the procedure to get the dispersion relation for the photon gas in the framework of DSR proposed and generalized by Magueijo and Smolin [5,6,7]. In Sec. 3, we derive the modified integration measure and the grand partition function of the photon gas. In Sec. 4, we study various thermodynamical quantities of the photon gas in details, and the cases $\lambda > 0$ and $\lambda < 0$ are compared with those of SR ($\lambda = 0$). In Sec. 5, we provide summaries of the paper.

2 Modified Dispersion Relation For Photon Gas

Theoretically, DSR itself cannot decide the dispersion relation used to describe the real nature, whereas only experiments can pick out the right formula [2,9]. Unfortunately, the current status of observations is incapable to draw a decisive conclusion. However, promising observations are emerging [13,14,15,21,22,23] and many possibilities to modify the dispersion relation in the framework of DSR have already been proposed.

In this paper, the model we adopt is DSR2, which was proposed and generalized by Magueijo and Smolin [5,6,7]. This model introduces a nonlinear modification to the action of the generators of Lorentz group in momentum space, and this renders the theory to be compatible with an observer-independent length scale. The modified boost generators are given as [5,6,7]

$$K^i = U^{-1}[p_0]L_0^i U[p_0], \quad (2)$$

where $L_{ab} = p_a \partial / \partial p^b - p_b \partial / \partial p^a$ are the standard generators of Lorentz group, and $U[p_0]$ is defined as

$$U[p_0](p_a) = \frac{p_a}{1 - \lambda p_0}, \quad (3)$$

with the parameter λ believed to be around the Planck length scale. From the above construction, with $U[p_0]$ defined to map the energy-momentum manifold onto itself,

$$U \circ (E, \vec{p}) = (f_1 E, f_2 \vec{p}), \quad (4)$$

any isotropic dispersion relation can be written in the following form [5,6,7],

$$E^2 f_1^2(E, \lambda) - p^2 f_2^2(E, \lambda) = m^2. \quad (5)$$

In this paper, we consider the case with $f_1 = 1$ and $f_2 = 1 + \lambda E$. The dispersion relation for the photon with zero mass becomes

$$p = \frac{E}{1 + \lambda E}. \quad (6)$$

When $\lambda > 0$, Eq. (6) implies a maximum momentum $p_{max} = \lambda^{-1}$, which is an invariant under deformed transformation laws. In contrast, $\lambda < 0$ corresponds to an energy upper bound for photons. Contrary to the modified SR in the framework of effective field theory where the dispersion relation for photons is spin-dependent, the dispersion relation is spin-independent in DSR.

Based on the relation (6), one obtains

$$\frac{dp}{dE} = \frac{1}{(1 + \lambda E)^2}. \quad (7)$$

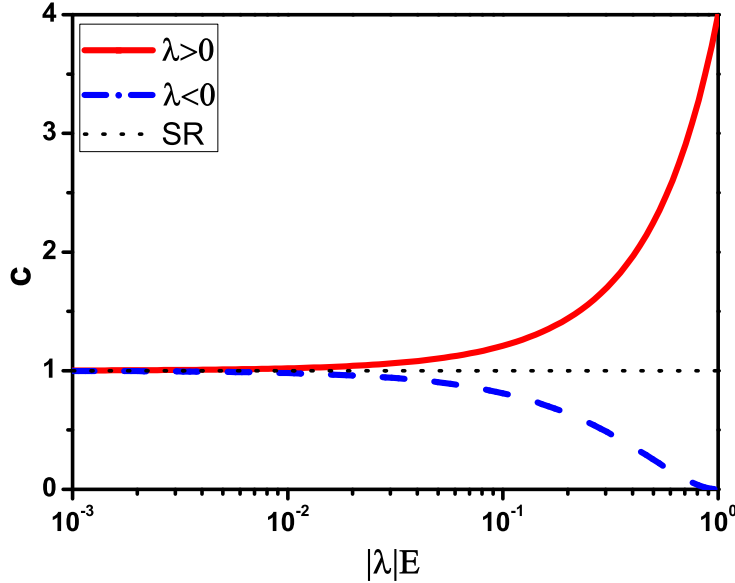


Fig. 1. The relation between the speed of light and the energy of photons. The red solid and blue dashed lines are results for scenarios where $\lambda > 0$ and $\lambda < 0$, respectively. The dotted horizontal line is the canonical result in the SR.

The speed of light is therefore energy-dependent,

$$c(E) = \frac{dE}{dp} = (1 + \lambda E)^2, \quad (8)$$

which is shown in Fig. 1. We can see explicitly that with energy increasing, the speed of light increases with a positive λ , and decreases with a negative one. If the theory is to provide a solution to the horizon problem without inflation, $\lambda > 0$ is required. However, we consider both signs in this paper, for the sake of completeness.

We have to emphasize that, as stressed by Amelino-Camelia [1,2,9], though many authors working on DSR insist on getting formulas that make sense all the way up to infinite particle energies, it is possible that DSR is relevant only at energy scales that are sub-Planckian (but nearly Planckian). If we actually expect to give up an intelligible picture of space-time and its symmetries above the Planck scale, then perhaps we should be open to the possibility of using mathematics that provides an acceptable (closed) logical picture of DSR only at the leading order (or some finite orders) in the Taylor expansion of formulas in powers of the Planck length. This possibility will not affect the results in most literatures, where only the effects at low energies were analyzed. However, in this paper, the integration in the phase space is up to E_P , so we need to assume that the formulas are tenable even up to the Planck scale in order to

be self-consistent.

3 Density Of Microstates and Grand Partition Function

There are possibilities of photon decays, *e.g.*, $\gamma \rightarrow e^+ + e^-$, when the dispersion relation (5) alone is considered with proper LV parameters and unmodified energy-momentum conservation laws [13,23]. However, these possibilities are totally invalid in DSR, since such kind of decays lead to energy thresholds for the decay of a single particle, and those are forbidden for the sake of relativistic principle [9]. Therefore it is proper to consider the photon gas composed from free particles forming a thermodynamical ensemble.

Consider the photon gas in a container of volume V . We assume that the momentum spectrum is continuous in the thermodynamical limit with an infinite V and negligible boundary conditions. Then the number of microstates available in the position ranging from \vec{r} to $\vec{r} + d\vec{r}$ and momentum ranging from \vec{p} to $\vec{p} + d\vec{p}$ is given by

$$dN = 2 \times \frac{1}{(2\pi\hbar)^3} d\vec{r} d\vec{p}, \quad (9)$$

where the factor 2 results from two directions of polarization.

3.1 Derivation Of Modified Integration Measure

The arguments reported in Refs. [4,26,27] imply that the net effect of a deformed measure of integration in DSR can be described with the replacement,

$$d^4p \rightarrow \theta(E) d^4p, \quad (10)$$

where $\theta(E)$ is a function of the energy E and needs to be adjusted by future insights on detailed analysis. Eq. (10) is often left out without proper justifications in previous works [29], and we would consider the deformation of the integration measure for completeness.

For the dispersion relation we adopt in (6), the law of composition of momenta is modified into

$$p \oplus \Delta p = (E(\vec{p}) + E(\Delta\vec{p}), \vec{p} + \theta(E)\Delta\vec{p}), \quad (11)$$

where Δp is an infinitesimal increment. Because of the virtue of relativistic principle, $p \oplus \Delta p$ satisfies the same functional form of Eq. (6). Consequently,

it is straightforward to get the form of $\theta(E)$,

$$\theta(E) = \frac{1}{(1 + \lambda E)^2}. \quad (12)$$

Thus for each spatial component p_i and a function $F'(p_i)$, which is the integrand of $F(p_i) = \int F'(p_i) dp_i$, the law of composition of momenta suggests that,

$$F'(p_i) = \lim_{\Delta p_i \rightarrow 0} \frac{F(p_i \oplus \Delta p_i) - F(p_i)}{\Delta p_i} = \frac{\partial F(p_i)}{\partial p_i} \frac{1}{(1 + \lambda E)^2}. \quad (13)$$

This in turn suggests that for one spatial momentum we have

$$F = \int F'(p_i) dp_i = \int \frac{\partial F(p_i)}{\partial p_i} \frac{1}{(1 + \lambda E)^2} dp_i, \quad (14)$$

which is equal to a replacement of the integration measure, $dp_i \rightarrow (1 + \lambda E)^{-2} dp_i$. In the case we are interested, with three spatial and one time dimensions, we finally get the modified integration measure,

$$d^4 p \rightarrow \frac{1}{(1 + \lambda E)^6} d^4 p. \quad (15)$$

It is worthy to mention that the form of the deformed measure of integration (15) is a little different from that used in Refs. [4,26,27]. Their results can be written in a united form as

$$d^4 p \rightarrow e^{-3\eta E/E_p} d^4 p. \quad (16)$$

If we assume that the modified dispersion relations adopted in our work and of earlier papers coincide in the leading-order approximation, we have

$$\lambda \sim \frac{\eta}{2E_p}, \quad (17)$$

and consequently,

$$e^{-3\eta E/E_p} - \frac{1}{(1 + \lambda E)^6} = e^{-6\lambda E} - \frac{1}{(1 + \lambda E)^6} \sim \mathcal{O}((\lambda E)^2). \quad (18)$$

We should emphasize that high-order differences for the integration measure result from different modified dispersion relations and mathematical frameworks. Nowadays, these differences do not really matter, because currently we cannot conclude which specific model of DSR should describe the real nature. Nevertheless, the leading-order corrections, which are of great scientific interests at the present stage, are the same in different models of DSR.

3.2 Derivation Of Modified Grand Partition Function

After we carefully consider the integration measure, we now turn to the grand partition function of a photon gas. The grand partition function Ξ of the photon gas in a container with fixed volume V is defined as

$$\ln \Xi = - \int \frac{8\pi V}{(2\pi\hbar)^3} p^2 \ln(1 - e^{-\frac{E}{k_B T}}) \frac{1}{(1 + \lambda E)^6} dp. \quad (19)$$

In the following we will always adopt the units in which $\hbar = k_B = 1$.

In the scenario where $\lambda > 0$, with relation between E and p given in Eqs. (6) and (7), the grand partition function is given in the form

$$\ln \Xi = - \int_0^\infty \frac{8\pi V}{(2\pi)^3} \frac{E^2}{(1 + \lambda E)^{10}} \ln(1 - e^{-\frac{E}{T}}) dE. \quad (20)$$

We adopt a dimensionless variant $x = E/T$ to simplify (20),

$$\ln \Xi = - \frac{VT^3}{\pi^2} \int_0^\infty \frac{x^2 \ln(1 - e^{-x})}{(1 + \lambda T x)^{10}} dx. \quad (21)$$

In the limit $\lambda \rightarrow 0$, it reduces to the normal SR result,

$$\ln \Xi = \frac{\pi^2}{45} VT^3. \quad (22)$$

The grand partition function Ξ of the photon gas when $\lambda < 0$ can be obtained similarly, and the result is

$$\ln \Xi = - \frac{VT^3}{\pi^2} \int_0^{-\frac{1}{\lambda T}} \frac{x^2 \ln(1 - e^{-x})}{(1 + \lambda T x)^{10}} dx. \quad (23)$$

The change of the upper limit in the integration results from the existence of a maximum energy $-1/(\lambda T)$. Still, it reduces to the normal SR result in the limit $\lambda \rightarrow 0^-$. We would like to point out that in this case, the integration diverges when T approaches to 1, and this implies that underlying quantum-gravitational theories should replace the DSR model when the temperature is around or larger than the Planck temperature. We observe that most models, in which an invariant energy or momentum scale exists with the speed of light decreasing with respect to energy, suffer from similar problems. In spite of this problem, we can mainly pay attention to the situation where T is lower bounded from 1, thus the theory is still well defined.

For both $\lambda > 0$ and $\lambda < 0$, we further set $\lambda = \pm 1$ respectively, and that is only a matter of units without losing generality. We write the grand partition

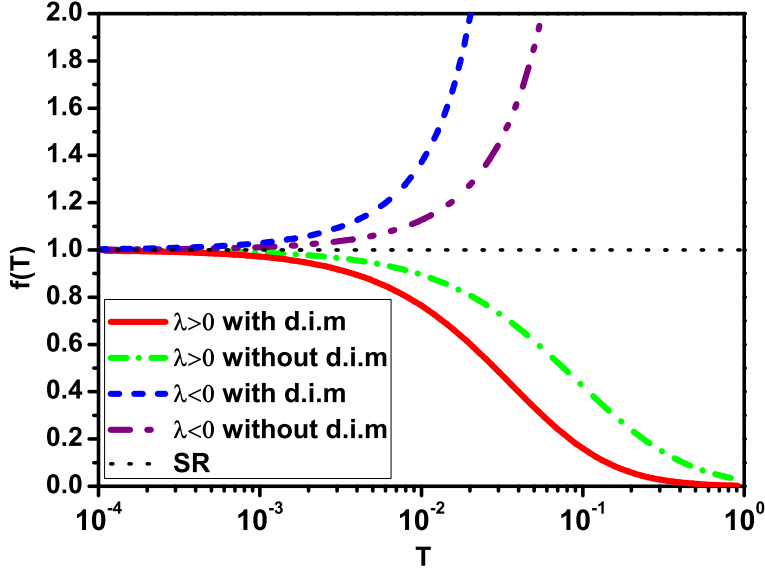


Fig. 2. The behavior of $f(T)$. The red solid line and the blue dashed line are the results of the situations in which $\lambda = 1$ and $\lambda = -1$ respectively. The green dash-dotted line and the purple dash-dot-dotted line are corresponding results without taking the deformed integral measure (d.i.m) into consideration. The dotted horizontal line is the canonical result in SR.

function Ξ in the following form,

$$\ln \Xi = \frac{\pi^2}{45} VT^3 f(T), \quad (24)$$

with

$$f(T) = -\frac{45}{\pi^4} \int_0^\infty \frac{x^2 \ln(1 - e^{-x})}{(1 + xT)^{10}} dx, \quad \lambda = 1, \quad (25)$$

and

$$f(T) = -\frac{45}{\pi^4} \int_0^{\frac{1}{T}} \frac{x^2 \ln(1 - e^{-x})}{(1 - Tx)^{10}} dx, \quad \lambda = -1, \quad (26)$$

for compact discussions below. Thus, all modifications from DSR are encoded in the function $f(T)$.

We show the behavior of $f(T)$ in Fig. 2. It is noteworthy to observe that, $f(T) < 1$ with $\lambda > 0$ while $f(T) > 1$ with $\lambda < 0$. Thus the number of available states changes according to λ . When λ is positive, the number of available states allowed is reduced, and while λ is negative, it is enhanced. This is the main result we get from thermodynamics of DSR. From Fig. 2, it is illustrated that the deformation of integration measure makes the changes more sharply. It should be carefully taken into account especially when T is large. At lower temperature $T \ll 1$, all lines meet the SR case with $f(T) = 1$.

4 Thermodynamic Properties

Having derived the expression of the partition function of the photon gas, we can study various thermodynamical properties in the following. Both the situations in which $\lambda > 0$ and $\lambda < 0$ are to be considered and compared, and we call them “positive-type theory” and “negative-type theory” respectively. As will be shown, all the thermodynamical variants depend merely on $f(T)$ and its derivatives. Furthermore, we have checked that the results presented here are almost the same even when some other forms of DSR models are considered instead of DSR2. Thus, our results are possible to represent general deformed dispersion relations qualitatively within DSR theories.

4.1 Internal Energy

The expression of the internal energy U for the photon gas is given by

$$U = T^2 \frac{\partial \ln \Xi}{\partial T} = \frac{\pi^2 V T^4}{15} \left(f + \frac{T}{3} f' \right), \quad (27)$$

where $f' = \frac{df(T)}{dT}$. Now the internal energy depends on both f and f' . We plot the internal energy U versus T in Fig. 3. It is shown that, compared to the case in SR, the internal energy decreases in the positive-type theory, and increases in the negative-type theory, and this can be roughly understood through the number of available states from Fig. 2. And the DSR results seem to be compatible with SR results when $T < 0.01$.

4.2 Specific Heat

The specific heat of the photon gas C_V is given as

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{4\pi^2 V T^3}{15} \left(f + \frac{2T}{3} f' + \frac{T^2}{12} f'' \right), \quad (28)$$

which now depends on f'' besides f and f' . We show the results in Fig. 4. Compared to the result in SR, the specific heat is remarkably smaller in the positive-type theory, and larger in the negative-type theory, when T approaches to 1. It is interesting to notice that in the positive-type theory, the specific heat tends to be a constant when T is large. The smaller C_V and larger C_V than that of SR in the positive-type and negative-type theories respectively reflect the situations in Fig. 3, where the energy increases slowly in positive-type theory and diverges in negative-type theory.

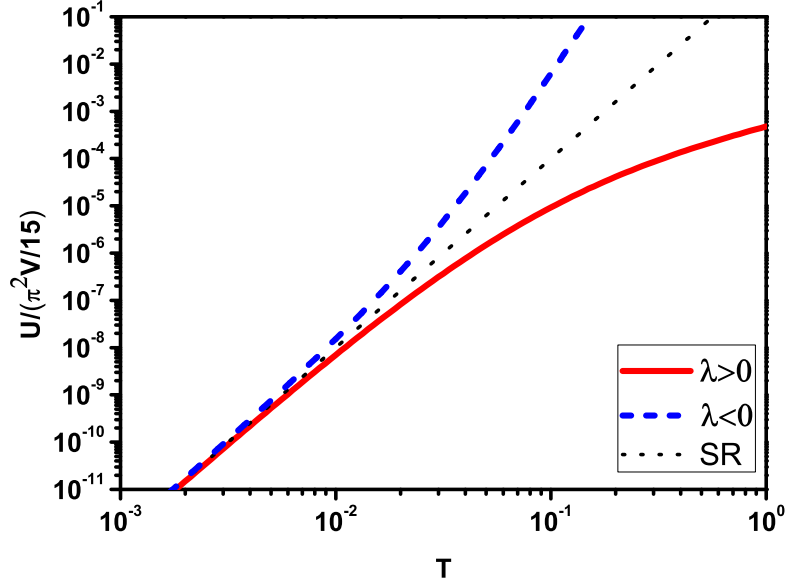


Fig. 3. The behavior of $U(T)$. The red solid line and the blue dashed line are the results of the situation in which $\lambda = 1$ and $\lambda = -1$ respectively. The dotted horizontal line is the canonical result in SR.

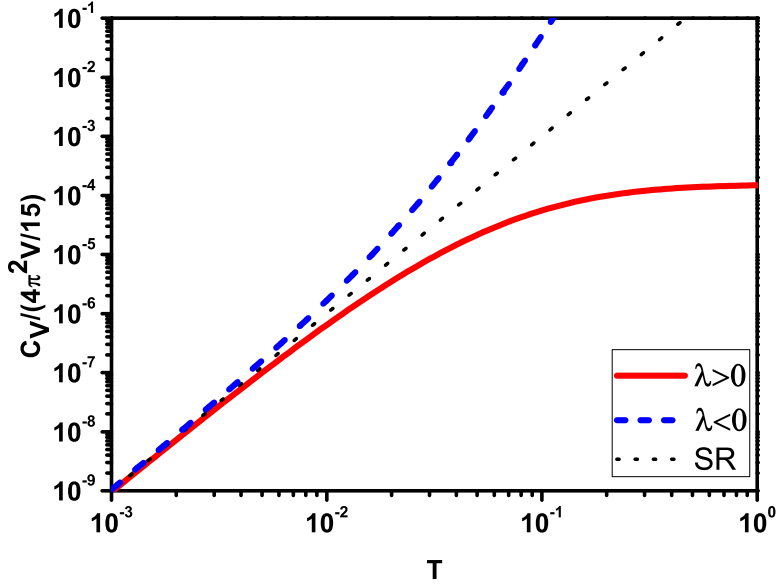


Fig. 4. The behavior of $C_V(T)$. The red solid line and the blue dashed line are the results of the situation in which $\lambda = 1$ and $\lambda = -1$ respectively. The dotted horizontal line is the canonical result in SR.

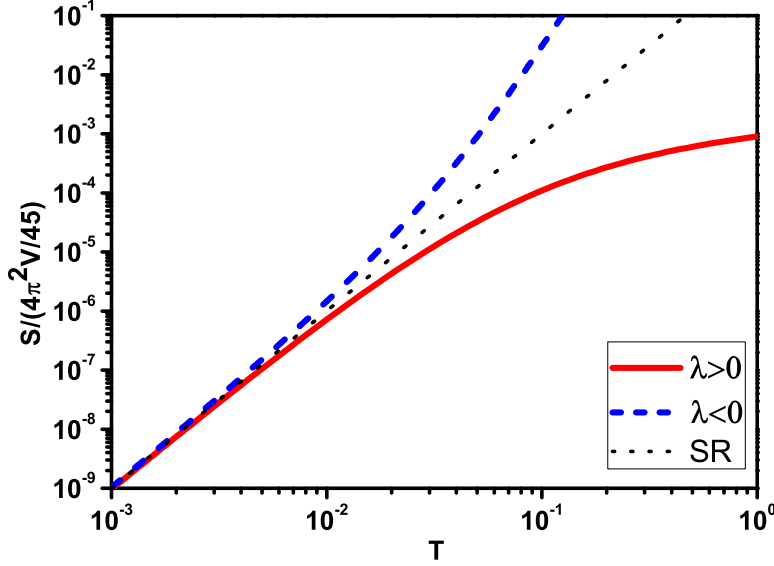


Fig. 5. The behavior of $S(T)$. The red solid line and the blue dashed line are the results of the situation in which $\lambda = 1$ and $\lambda = -1$ respectively. The dotted horizontal line is the canonical result in SR.

4.3 Entropy

We can also obtain the interesting thermodynamical quantity, entropy S ,

$$S = \left(\ln \Xi + T \frac{\partial \ln \Xi}{\partial T} \right) = \frac{4\pi^2 V T^3}{45} \left(f + \frac{T}{4} f' \right), \quad (29)$$

which now, like the internal energy U , depends on f and f' . The result of the relation between entropy and temperature is illustrated in Fig. 5. It is clearly seen that the entropy grows much slower in the positive-type theory and faster in the negative-type theory, and this is consistent with our previous observations. In fact, the different behaviors of the entropy of the system for two types of theories result directly from different modifications of the total available number of microstates. When $\lambda > 0$, there are fewer available states, hence less chaos, while $\lambda < 0$ induces more available states and more chaos. The entropy measures chaos naturally. Thus the behavior of the entropy is rather understandable.

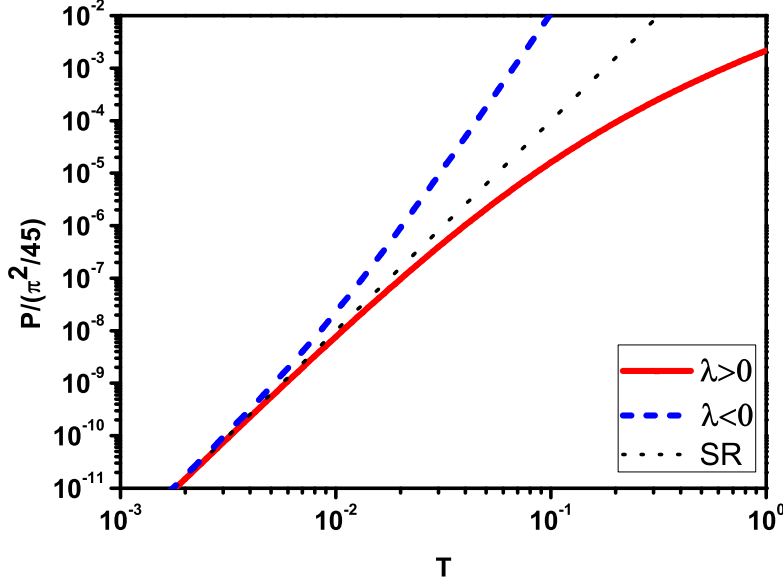


Fig. 6. The behavior of $P(T)$. The red solid line and the blue dashed line are the results of the situation in which $\lambda = 1$ and $\lambda = -1$ respectively. The dotted horizontal line is the canonical result in SR.

4.4 Pressure and Pressure-Energy Density Relation

The expression of Ξ can be used to get the pressure P of the photon gas

$$P = T \frac{\partial \ln \Xi}{\partial V} = \frac{\pi^2}{45} T^4 f(T), \quad (30)$$

which appears merely a function of f . The relation between the pressure P and T is given in Fig. 6. We can see that the pressure reduces when $\lambda > 0$, with respect to the case in SR, and reverses when $\lambda < 0$. It is understandable that the fewer number of states ($\lambda > 0$) causes the photons to gather with an effective attractive force, while the more number of density induces an effective repulsive force, compared to the case in SR. Such an effective force is important when the dynamical behavior of photons is under consideration.

Although one cannot directly detect the internal energy U of the photon gas, the relation between the pressure P and the energy density $\rho = U/V$ is measurable experimentally and is considered to be important to various aspects. The modifications of P/ρ , in principle, are effects appearing at the macroscopical level and can have significant influences on the early universe. We derive the relation,

$$\frac{P}{\rho} = \frac{1}{3 + T \frac{f'}{f}}, \quad (31)$$

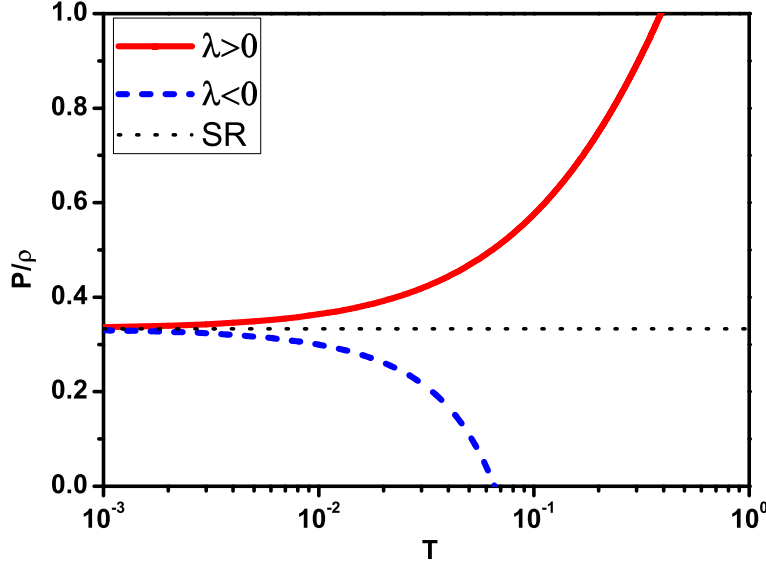


Fig. 7. The behavior of P/ρ . The red solid line and the blue dashed line are the results of the situation in which $\lambda = 1$ and $\lambda = -1$ respectively. The dotted horizontal line is the canonical result in SR.

whose modification now depends on the ratio of f' and f . We plot P/ρ via T in Fig. 7. It is clearly seen that in the ultra-relativistic regime for photons, the well-known canonical relation, $P = \rho/3$, has to be modified. In contrast with our naive expectations, P/ρ with a positive λ is larger than that with a negative one. It is caused by the negative slope of $f(T)$ when $\lambda > 0$, and the positive slope of $f(T)$ when $\lambda < 0$.

5 Conclusion

In this paper, we derived the grand partition function of a photon gas in doubly special relativity (DSR), based on careful considerations of the deformed dispersion relation and the modified measure of integration, as well as the upper bound of energy/momentum. Then we discussed the thermodynamical quantities of the photon gas and showed that the behaviors of thermodynamical variants are modified according to the deformation parameter λ .

These modifications are not remarkable when the temperature is low, say $T < 10^{-3}E_P$, thus it is not easy to detect the effects directly in present laboratory experiments. However, significant differences exist when the energy approaches to the Planck scale. Therefore, it is suggested that these modifications play an important role in cosmology, especially on properties of the early universe.

Thus if these modifications turn out to be correct, the behavior of photons in the early stage of the Big Bang will be very different from what we observe at low energy. For example, the modification of the ratio of pressure to energy density we have considered in Sec. 4.4 leads to a different characteristic of the evolution of the weight that radiations occupy among all types of energy in the whole universe, and its consequence is of noticeable importance on the evolution of the universe.

In summary, we derived the thermodynamics of the photon gas in the framework of DSR. We carefully included effects from the modified energy-momentum dispersion relation, the deformed integration measure, and the upper bound of energy/momentum, and some of them are often left out by other studies. It is shown in detail that different behaviors, other than those in special relativity, emerge when the energy approaches to the quantum gravity scale. Thus, the results could have significant consequences on the early universe, such as the inflation, in cosmological study.

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References

- [1] G. Amelino-Camelia, Phys. Lett. B **510** (2001) 255 [arXiv:hep-th/0012238].
- [2] G. Amelino-Camelia, Int. J. Mod. Phys. D **11** (2002) 35 [arXiv:gr-qc/0012051].
- [3] J. Kowalski-Glikman, Phys. Lett. A **286** (2001) 391 [arXiv:hep-th/0102098].
- [4] J. Kowalski-Glikman, Phys. Lett. A **299** (2002) 454 [arXiv:hep-th/0111110].
- [5] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88** (2002) 190403 [arXiv:hep-th/0112090].
- [6] J. Magueijo, L. Smolin, Class. Quant. Grav. **21** (2004) 1725-1736 [gr-qc/0305055].
- [7] J. Magueijo and L. Smolin, Phys. Rev. D **67** (2003) 044017 [arXiv:gr-qc/0207085].

- [8] G. Amelino-Camelia, Int. J. Mod. Phys. D **11** (2002) 1643 [arXiv:gr-qc/0210063].
- [9] G. Amelino-Camelia, Symmetry **2** (2010) 230 [arXiv:1003.3942 [gr-qc]].
- [10] G. Amelino-Camelia, M. Arzano, Y. Ling and G. Mandanici, Class. Quant. Grav. **23** (2006) 2585 [arXiv:gr-qc/0506110].
- [11] G. Salesi and E. Di Grezia, Phys. Rev. D **79** (2009) 104009 [arXiv:0902.3763 [gr-qc]].
- [12] L. Shao and B.-Q. Ma, arXiv:1006.3031 [hep-th].
- [13] D. Mattingly, Living Rev. Rel. **8** (2005) 5 [arXiv:gr-qc/0502097].
- [14] G. Amelino-Camelia, arXiv:0806.0339 [gr-qc].
- [15] S. Liberati and L. Maccione, Ann. Rev. Nucl. Part. Sci. **59** (2009) 245 [arXiv:0906.0681 [astro-ph.HE]].
- [16] Z. Xiao, L. Shao, B.-Q. Ma, Eur. Phys. J. C **70** (2010) 1153 [arXiv:1011.5074 [hep-th]].
- [17] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Nature **393** (1998) 763 [arXiv:astro-ph/9712103].
- [18] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. A **12** (1997) 607 [arXiv:hep-th/9605211].
- [19] J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, Astrophys. J. **535** (2000) 139 [arXiv:astro-ph/9907340].
- [20] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **63** (2001) 124025 [arXiv:hep-th/0012216].
- [21] Z. Xiao and B.-Q. Ma, Phys. Rev. D **80** (2009) 116005 [arXiv:0909.4927 [hep-ph]].
- [22] L. Shao, Z. Xiao and B.-Q. Ma, Astropart. Phys. **33** (2010) 312 [arXiv:0911.2276 [hep-ph]].
- [23] L. Shao, B.-Q. Ma, Mod. Phys. Lett. A **25** (2010) 3251 [arXiv:1007.2269 [hep-ph]].
- [24] S. Alexander, R. Brandenberger and J. Magueijo, Phys. Rev. D **67** (2003) 081301 [arXiv:hep-th/0108190].
- [25] L. Mersini, M. Bastero-Gil and P. Kanti, Phys. Rev. D **64** (2001) 043508 [arXiv:hep-ph/0101210].
- [26] G. Amelino-Camelia and S. Majid, Int. J. Mod. Phys. A **15** (2000) 4301 [arXiv:hep-th/9907110].

- [27] G. Amelino-Camelia, N. Loret, G. Mandanici and F. Mercati, arXiv:0906.2016 [gr-qc].
- [28] A. Camacho and A. Macias, Gen. Rel. Grav. **39** (2007) 1175 [arXiv:gr-qc/0702150].
- [29] S. Das and D. Roychowdhury, Phys. Rev. D **81** (2010) 085039 [arXiv:1002.0192 [hep-th]].